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Universal quark-lepton mixing and determination of neutrino masses

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Abstract

If three right-handed neutrinos are added to the Standard Model, then, for the three known generations, there are six quarks and six leptons. It is then natural to assume that the symmetry considerations that have been applied to the quark matrices are also valid for the lepton mass matrices. Under this assumption, the solar and atmospheric neutrino data can be used to determine the individual neutrino masses. Three minima have been found, using the χ^2 fit, and, from these minima, it is determined that the mass of the lightest neutrino is 1.3×10^{-3} eV, that of the next heavier neutrino is 1.3×10^{-2} eV, while the mass of the heaviest neutrino is 3.4×10^{-2}, 5.8×10^{-2} or 9.4×10^{-2} eV.

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1   Introduction

In the work of Lehmann, Newton, and Wu [1], the Kobayashi-Maskawa [2] matrix is expressed in terms of the masses of the three generations of quarks:

$$\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix}. \tag{1.1}$$

This is accomplished by introducing a new horizontal symmetry. Some of the earlier attempts in this direction are given in [3, 4].

Recent experiments at Super-Kamiokande [5–7] indicate the presence of neutrino oscillations, which would imply that the neutrinos are not all massless. If it is accepted that the neutrinos are not massless, then it is most natural in the Standard Model [8] to introduce three right-handed neutrinos in addition to the three known left-handed ones. In this way, there are six quarks and six leptons.[1] In this paper, the consequences of a universal quark-lepton mixing are studied. In other words, the method of [1] is used to express the lepton KM matrix and the neutrino mixing matrix in terms of the masses of the three generations of leptons:

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix} \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}. \tag{1.2}$$

Of course the masses of the three charged leptons are accurately known, leaving as unknown parameters the masses of the three neutrinos. Thus there are three parameters to be determined instead of seven, the three masses plus the four in the lepton KM matrix.

It is the purpose of this paper to use the data from solar neutrinos [10, 11, 12, 6] and atmospheric neutrinos [5, 7] to determine the three neutrino masses separately, not only the differences of their squares.

2   The mixing matrix

The quark mixing matrix proposed by Lehmann et al. [1] relates the mixing to the actual quark masses. These mass matrices are each given by four parameters a, b, c, and d, and, when applied to the leptons without modification, take the form

$$M(\ell) = \begin{pmatrix} 0 & d(\ell) & 0 \\ d(\ell) & c(\ell) & b(\ell) \\ 0 & b(\ell) & a(\ell) \end{pmatrix}, \tag{2.1}$$

$$M(\nu) = \begin{pmatrix} 0 & id(\nu) & 0 \\ -id(\nu) & c(\nu) & b(\nu) \\ 0 & b(\nu) & a(\nu) \end{pmatrix}, \tag{2.2}$$

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[1]As shown in [9], neutrino-oscillation experiments cannot distinguish between massive Majorana and Dirac neutrinos.

with

$$b^2(\ell) = 8c^2(\ell), \qquad b^2(\nu) = 8c^2(\nu). \tag{2.3}$$

The diagonalization of these mass matrices [13] is achieved by the orthogonal matrices $R(\ell)$ and $R(\nu)$, where

$$M(\ell) = R(\ell)M_{\text{diag}}(\ell)R^{\text{T}}(\ell), \tag{2.4}$$

$$M(\nu) = \text{diag}(-i, 1, 1)R(\nu)M_{\text{diag}}(\nu)R^{\text{T}}(\nu)\text{diag}(i, 1, 1). \tag{2.5}$$

If there is CP violation in the lepton sector, then the imaginary entries in (2.2) and (2.5) are required. If by any chance there is no CP violation in the lepton sector, then both i and $-i$ there should be replaced by 1.

The diagonal mass matrices have the form

$$M_{\text{diag}} = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix} = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & -m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}, \tag{2.6}$$

where $\lambda_2 < 0$ and

$$m_1 \leq m_3. \tag{2.7}$$

We shall here only be concerned with $M(\nu)$ and $R(\nu)$.

In dealing with quarks [1], the observed quark masses allow a much stronger form for the inequality (2.7), namely

$$m_1 \leq m_2 \leq m_3. \tag{2.8}$$

The lack of direct experimental data on the neutrino masses implies that (2.7) can be used, but not (2.8). The first task is therefore to determine the allowed region in the space (m_1, m_2, m_3), which must be between those permitted by (2.7) and (2.8).

The parameters a, b, c, and d are related to the masses by the following conditions,

$$\begin{aligned} a + c &= S_1 = m_3 - m_2 + m_1, \\ 8c^2 + d^2 - ac &= -S_2 = m_3m_2 - m_3m_1 + m_2m_1, \\ ad^2 &= -S_3 = m_1m_2m_3. \end{aligned} \tag{2.9}$$

The cubic equation for the parameter a is then

$$9a^3 - 17S_1a^2 + (8S_1^2 + S_2)a - S_3 = 0. \tag{2.10}$$

Any real cubic equation can have either one or three real solutions. Where there is *one* real solution, that one is negative, and thus unphysical, as is seen from (2.9). Where there are *three* real solutions, one of them is negative, while two are positive. We shall refer to these two positive solutions as Solution 1 (larger a) and Solution 2 (smaller a).

These considerations can be used to determine the allowed physical region in the $(m_1/m_3, m_2/m_3)$ plane, as shown in Fig. 1. This region is only slightly larger than the triangle given by the inequality (2.8), with two additional regions, one where $m_2 > m_3$ and the other a very small one with $m_1 > m_2$.
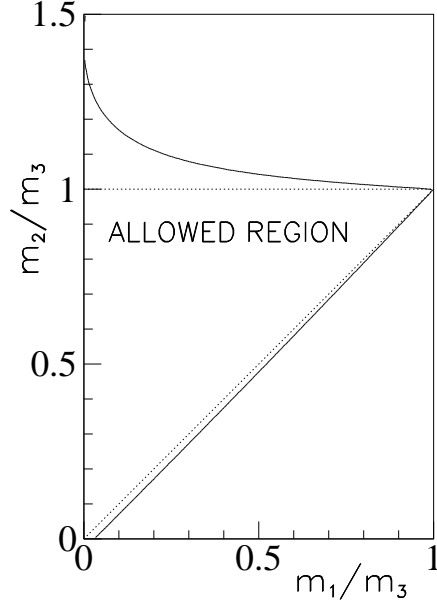
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Figure 1: The allowed region for the three neutrino masses m_1, m_2 and m_3 is within the *solid* contour.

3   The three-family MSW mechanism

The coupled equations satisfied by the three neutrino wave functions are [14]

$$
i \frac{\mathrm{d}}{\mathrm{d}r} \begin{pmatrix} \phi_1(r) \\ \phi_2(r) \\ \phi_3(r) \end{pmatrix} = \left[\begin{pmatrix} D(r) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \frac{1}{2p} \begin{pmatrix} M_{11}^2 & M_{12}^2 & M_{13}^2 \\ M_{21}^2 & M_{22}^2 & M_{23}^2 \\ M_{31}^2 & M_{32}^2 & M_{33}^2 \end{pmatrix} \right] \begin{pmatrix} \phi_1(r) \\ \phi_2(r) \\ \phi_3(r) \end{pmatrix}, \tag{3.1}
$$

where $D(r) = \sqrt{2}\, G_{\mathrm{F}} N_e(r)$, with G_{F} the Fermi weak-interaction constant and $N_e(r)$ the solar electron density at a distance r from the center of the sun. Furthermore, we denote $\nu_e = \phi_1$, $\nu_\mu = \phi_2$, $\nu_\tau = \phi_3$.

The evolution of the neutrino wave functions is determined by the squared mass matrix,

$$
[M(\nu)]^2 = \begin{pmatrix} d^2 & cd & bd \\ cd & b^2 + c^2 + d^2 & b(a+c) \\ bd & b(a+c) & a^2 + b^2 \end{pmatrix} \equiv \begin{pmatrix} M_{11}^2 & M_{12}^2 & M_{13}^2 \\ M_{21}^2 & M_{22}^2 & M_{23}^2 \\ M_{31}^2 & M_{32}^2 & M_{33}^2 \end{pmatrix}, \tag{3.2}
$$

the neutrino momentum, p, and the solar electron density. Here, $M_{ij}^2 \equiv (M^2)_{ij}$. The eigenvalues of the squared mass matrix (multiplied by $r_0/2p$, with r_0 defined below) are denoted μ_1, μ_2, and μ_3, and ordered such that

$$
\mu_1 \leq \mu_2 \leq \mu_3. \tag{3.3}
$$

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It is actually a good approximation to take an exponential electron density, $N_e(r) = N_e(0) \exp(-r/r_0)$. A fit to the solar density as given by [15] leads to $r_0 = 6.983 \times 10^4$ km. For this case of an exponential solar density, the three-component wave equation can be solved in terms of generalized hypergeometric functions, $_2F_2$ [16].

We scale and shift the radial variable, $u = r/r_0 + u_0$, with u_0 determined such that

$$D(0)r_0 e^{u_0} = 1. \tag{3.4}$$

The above equation (3.1) may then be written as

$$i\frac{\mathrm{d}}{\mathrm{d}u}\begin{pmatrix}\psi_1\\\psi_2\\\psi_3\end{pmatrix} = \begin{pmatrix}\omega_1 + e^{-u} & \chi_2 & \chi_3\\ \chi_2 & \omega_2 & 0\\ \chi_3 & 0 & \omega_3\end{pmatrix}\begin{pmatrix}\psi_1\\\psi_2\\\psi_3\end{pmatrix}, \tag{3.5}$$

where ω_2 and ω_3 denote the eigenvalues of the 2×2 mass matrix involving ψ_2 and ψ_3.

With $x = e^{-u}$, one finds for ψ_1 a differential equation of the form

$$\left[\prod_{j=1}^{3}\left(x\frac{\mathrm{d}}{\mathrm{d}x} - i\mu_j\right) - ix\prod_{j=2}^{3}\left(x\frac{\mathrm{d}}{\mathrm{d}x} - i\omega_j + 1\right)\right]\psi_1 = 0 \tag{3.6}$$

the solutions of which,

$$\psi_1(u) = \sum_{j=1}^{3} C_j \psi_1^{(j)}(u), \tag{3.7}$$

can be given in terms of generalized hypergeometric functions $_2F_2$ as

$$\psi_1^{(j)}(u) = e^{-i\mu_j u}\,_2F_2\left[\begin{matrix}1 - i(\omega_2 - \mu_j), & 1 - i(\omega_3 - \mu_j)\\ 1 - i(\mu_k - \mu_j), & 1 - i(\mu_\ell - \mu_j)\end{matrix}\middle| ie^{-u}\right] \tag{3.8}$$

with $k, \ell = 1, 2, 3$, $k \neq j$, $\ell \neq j$. For the other flavours, $\psi_2(u)$ and $\psi_3(u)$ are given by similar expressions, with shifted parameters.

There is not much information about these functions $_2F_2$ in the literature [17]. For the case of two flavours, the products in (3.6) go only up to $j = 2$, and a familiar confluent hypergeometric function $_1F_1$ (also denoted Whittaker function or parabolic cylinder function) is obtained [18].

In order to impose the boundary conditions that only *electron* neutrinos are produced in the sun, we have to determine these functions at large and negative values of u. The series expansion is in principle convergent, but it is not practical for large absolute values of both parameters and the argument. Instead, methods have been developed for the evaluation by a combination of relating them to $_3F_1$ and asymptotic methods [16].

4   Results

Since the only unknown parameters for the present theory of universal quark-lepton mixing are the three neutrino masses, it remains to determine these masses using experimental data on solar and atmospheric neutrinos.

For the atmospheric neutrinos we take the 8 data points for ν_μ and the 8 data points for ν_e, as reported recently [5]. These sixteen data points are treated as separate inputs, but we allow two overall normalization constants for the two sets of data.

For the solar-neutrino data, we use the total rates from the Chlorine experiment [10], the Gallium experiments [11, 12] (we average the two results) and the Super-Kamiokande experiment [6]. We adopt the neutrino energy spectra and detector efficiencies as given by Bahcall et al., and, for the latter detector, we also include the neutral-current cross section [19].

The determination of the three neutrino masses is to be carried out by a χ^2 fit to these pieces of data. Since the number of degrees of freedom is 14, a good χ^2 fit would give support to this idea that the mass mixing is universal for quarks and leptons. For this purpose, we have scanned the entire (m_1, m_2, m_3) parameter space. It is the necessity to cover this entire space that makes it essential to develop the theory described in Sec. 3 here.

It is found that there are three minima. They are:

(A) Solution 1:	(B) Solution 2:	(C) Solution 2:
$m_1 = 0.0016$ eV	$m_1 = 0.0011$ eV	$m_1 = 0.0011$ eV
$m_2 = 0.013$ eV	$m_2 = 0.013$ eV	$m_2 = 0.013$ eV
$m_3 = 0.034$ eV	$m_3 = 0.058$ eV	$m_3 = 0.094$ eV
with $\chi^2 = 31$	with $\chi^2 = 23$	with $\chi^2 = 20$

A few simple conclusions can be drawn from this set of mass values. First, with 14 degrees of freedom, the three values of χ^2 must be considered to be quite good. *This is evidence in favour of universal quark-lepton mixing.* In our mind the difference in these three values of χ^2 is not significant. Secondly, the three values for m_2 are the same—a surprise to us. This presumably means that the relative accuracy in the present determination of m_2 is better than those of m_1 and m_3. This can be understood in the following way. The relative accuracy for m_1 is low because m_1 itself is quite small, and the value of χ^2 is not very sensitive to such small masses. The average of these three m_1 is given in the abstract, and very roughly we guess the error to be about a factor of two. On the other hand, the uncertainly in the value of m_3 is simply due to the three minima giving different values. It is a matter of which one of the three minima is physically the correct one.

For completeness, we give the rotation matrix $R(\nu)$. The Euler angles (θ, ϕ, ψ) [20] are: $(22°, 28°, -8°)$ for (A), $(35°, 21°, -5°)$ for (B), and $(31°, 19°, -3°)$ for (C).

To demonstrate the scanning of the entire region of allowed values of neutrino masses, we show in Fig. 2 an example of such a scan showing contours of constant χ^2. In this

figure, the value of m_3 used is 0.058 eV, corresponding to the minimum (B) above. It is from such scans that we know there are only three minima. In this figure, some minor irregularities along the edges of the allowed region are artifacts of the finite grid spacing.
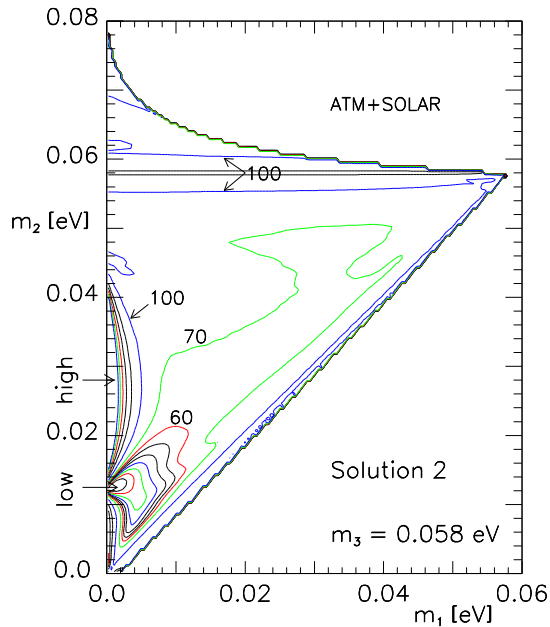

Figure 2: Fits to the atmospheric- and solar-neutrino data. Contours are shown at $\chi^2 = 30$, $35, \ldots, 60, 70, 100, 150, \ldots, 350$. (The outer contour outlines the boundary of the allowed region, cf. Fig. 1.)

It is difficult to compare the present result with the previously given allowed regions in the parameter space. The reason is that the allowed regions have typically been given on the basis of the mixing of two neutrino species, while there is significant mixing among all three neutrinos, or the studies concentrate on *either* solar *or* atmospheric neutrinos [21, 22]. The following comparisons are nevertheless of interest.

(a) It is probably correct to compare the $m_3^2 - m_2^2$ here with the δm^2 from atmospheric-neutrino data. In this case, the $m_3^2 - m_2^2$ for the three minima covers roughly the allowed range of δm^2, with minimum (B) near the center, and minima (A) and (C) near the edges of the allowed region.

(b) It is probably not too far wrong to identify $m_2^2 - m_1^2$ here with the δm^2 from solar-neutrino data. If so, the values are reasonably close, except with the so-called "low-mass–low-probability" (LOW) solution [21].

(c) It is more difficult to discuss the strength of the coupling. It can nevertheless be concluded, in the context of solar neutrinos, that the present solutions are significantly closer to the "large-mixing-angle" (LMA) solution than to the "small-mixing-angle" (SMA) solution.

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In connection with the problem of distinguishing between the three minima, more accurate data from Super-Kamiokande and related experiments are needed. Also, a better understanding of the high-energy *hep* neutrino flux [21] could make the electron recoil energy spectrum useful for this purpose. Furthermore, important information may be forthcoming also from the long-baseline experiments [23] and exotic atom experiments [24].

Acknowledgments

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